Homework Assignment #4

Assigned on October 11, 2022

Due on Canvas October 26, 2022

by 11:59PM

50 points

Dieudonne Muhirwa M14185704

Allen Detmer M15216736

JiaBin Chang M13782945

Joseph Matawaran M15228497

This is a ‘paper-and-pencil’ assignment. Please type your solutions

**Problem 1.** (10 points) Let r, e, and c be propositional letters as follows:

r: “a person is a radical”,

e: “a person is electable”,

c: “a person is conservative”

Consider the following assertion **“a person who is a radical is electable if he/she is conservative, but otherwise the person is not electable”.**

(a) Which of the following are the correct representations of this assertion?

Use &, V, ~, →, ←→ to stand for conjunction, disjunction, negation, implication and bidirectional implication respectively.

1. (r & e) ←→c

Incorrect representation because this implies that the person is conservative if and only if they are radical and electable but the person does not have to be radical and electable to be conservative.

1. r → (e ←→ c)

Correct representation because it implies the same as the given assertion. If the person is radical then they are electable if and only if they are conservative.

1. r →((c → e) V ~e)

The representation is incorrect because it implies that if a person is radical, then if they are conservative, then they are electable or else they are not electable. This representation fits the assertion and is always true for all interpretations, therefore, this is not a correct assertion.

(b) Which sentence in (a) can be expressed as a Horn clause? Explain your answer.

As discussed in class, horn clause is a disjunction with at most one positive literal. As we can see below all sentences in (a) can be expressed in their Horn clause representation because all of them satisfy the definition of horn clause of having at most one positive literal.

1. (r & e) ←→c ≡ ((r & e) → c) & (c → (r & e))

≡ ((r & e) → c) & (c → r) & (c → e)

1. r → (e ←→ c) ≡ r → ((e → c) & (c → e))

≡ ~r V ((~e V c) & (~c V e))

≡(~r V ~e V c) & (~r V ~c V e))

1. r →((c → e) V ~e): *True → True*

**Problem 2.** (10 points) Consider the following argument, where food, drinks, party denote propositions.

[(food → party) V (drinks → party)] |= [(food & drinks) → party]

1. Use truth tables to determine whether the argument is Valid

p = food  
q = drinks  
r = party

[(p -> r) V (q -> r)] := [(p^q) -> r]

| p | q | r | p->r (premise) | q->r (premise) | (p → r) V (q->r)  (premise) | p ^ q | p ^ q → r | [(p->r) V= V (q->r)] p^q -> r (conclusion) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| T | T | T | T | T | T | T | T | T |
| T | T | F | F | F | F | T | F | T |
| T | F | T | T | T | T | F | T | T |
| T | F | F | F | T | T | T | T | T |
| F | T | T | T | T | T | F | T | T |
| F | T | F | T | F | T | F | T | T |
| F | F | T | T | T | T | F | T | T |
| F | F | F | T | T | T | F | T | T |

1. Convert the left hand-side and the right hand-side in CNF (conjunction of clauses/disjunctions) and show how the results confirm your answer in part (a)

[(p -> r) V (q -> r)] := [(p^q) -> r]

[(~p V r) V (~q V r)] := [~(p V q) V r]

Left Side: (p -> r) V (q -> r)

= (~p V r) V (~q V r)

= (~p V ~q) V r

= ~(p ^ q) V r

= (p ^ q) -> r

Right side: ~(p V q) V r

= (p^q) -> r

Since P->P, so it is valid for any P

1. Prove your answer to (a) using resolution.

To show that KB |= α, we show that (KB ∧¬α) is unsatisﬁable, which means empty clause.

[(p -> r) V (q -> r)] := [(p^q) -> r]

KB = (p -> r) V (q -> r)

α = [(p^q) -> r]

KB ^ ~α

=>[(p -> r) v (q -> r)] ^ ~[(p^q) -> r]

=>[(~p v r) v (~q v r)] ^ ~[~(p ^ q) v r]

=>[~p v ~q v r] ^ (p ^ q) ^ r

=> empty clause

Therefore KB |= α is valid.

**Problem 3.** (10 points) Consider the following argument, where food, drinks, party denote propositions.

[(food → party) V (drinks → party)] |- [(food & drinks) → party]

Investigate the validity of this argument using syntactic proofs.

p = food  
q = drinks  
r = party

| Premises | Premises ID | Formula | Premises and rules used | Remarks |
| --- | --- | --- | --- | --- |
| Prem | 1 | (p → r) v (q → r) |  | This is the 1st premise of the argument |
| 1 | 2 | (~p v r) v (~q v r) | 1 Material Implication | Rewrite (p → r) to (~p v r) and (q → r) to (~q v r) |
| 1,2 | 3 | (~p v r) v (r v ~q) | 2 Associative property | Rearrange the sequence from (~p v r) v (~q v r) to (~p v r) v (r v ~q) |
| 1,2,3 | 4 | ~p v r v ~q | 3 tautology | Since r v r |- r, (~p v r) v (r v ~q) |- ~p v r v ~q |
| 1,2,3,4 | 5 | (~p v ~q) v r | 4 Associative property | Rearrange the sequence from (~p v r v ~q) to (~p v ~q) v r |
| 1,2,3,4,5 | 6 | ~(p ^ q) v r | 3 DeMorgan rules | From DeMorgan rules, (~p v ~q) |- ~(p ^ q) |
| 1,2,3,4,5,6 | 7 | (p ^ q) → r | 5 Material Implication | Rewrite ~(p ^ q) v r to (p ^ q) → r and get the conclusion |

**Problem 4.** (5 points) Consider now the sentence, where food, drinks, party denote propositions.

[(food → party) V (drinks → party)] → [(food & drinks) → party]

Use truth tables to investigate whether this sentence is

1. Satisfiable (i.e., there exists an assignment of truth values which make it true)

2. Unsatisfiable (i.e., there is no assignment of truth values which make it true)

**Answer.**

[(food → party) V (drinks → party)] → [(food & drinks) → party]

Let:

Food: F

Drink: D

Party: P

Simplified form:

[(F → P) V (D → P)] → [(F & D) → P]

**Truth Table.**

| F | D | P | F->P | D->P | F ^ D | (F → P) V (D->P) | F ^ D → P | [(F->P) V(D->P)]→ [( F^D) -> P] |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| T | T | T | T | T | T | T | T | T |
| T | T | F | F | F | T | F | F | T |
| T | F | T | T | T | F | T | T | T |
| T | F | F | F | T | F | T | T | T |
| F | T | T | T | T | F | T | T | T |
| F | T | F | T | F | F | T | T | T |
| F | F | T | T | T | F | T | T | T |
| F | F | F | T | T | F | T | T | T |

The sentence is satisfiable as there exists an assignment of truth value which makes it true. Hence, proven by the truth table.

**Problem 4.** (15 points)

Suppose you are given the following axioms:

1. 0 ≤ 4.

2. 6 ≤ 8.

3. ∀x x≤ x.

4. ∀x x≤ x + 0.

5. ∀x x+ 0 ≤ x.

6. ∀ x, y x + y ≤ y + x.

7. ∀ w, x, y, z w ≤ y ∧ x ≤ z ⇒ w + x ≤ y + z.

8. ∀ x, y, z x ≤ y ∧ y ≤ z ⇒ x ≤ z

(a) Give a **backward-chaining** proof of the sentence 6 ≤ 4 + 8. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that lead to success, not the irrelevant steps.

### **ANSWER**

6 <= 4 + 8

x = 6+0, z = 4+8, y = 6

S(8) {(x/6+0), (y/6), (z/(4+8)} 6+0 <= 6 ∧ 6 <= (4 + 8) ⇒ (6+0) <= 4 + 8

S(6) {(x/4), (y/8)} 4 + 8 <= 8 + 4

S(7) {(w/6),(x/0),(y/8),(z/4)} 6 <= 8 ∧ 0 <= 4 ⇒ 6 + 0 <= 8 + 4

S(1) 0 <= 4 S(2) 6 <= 8

(b) Give a **forward-chaining** proof of the sentence 6 ≤ 4 + 8. Again, show only the steps that lead to success.

### **ANSWER**

Prove: 6 <= 4 + 8

S(7){(w/6),(x/0),(y/8),(z/4)} 6 <= 8 ∧ 0 <= 4 ⇒ 6 + 0 <= 8 + 4

S(6) {(x/8),(y/4)} 8 + 4 <= 4 + 8

S(4) {x/6} 6 <= 6 + 0

S(8){(x/6+0), (y/8+4), (z/4+8)} 6+0 <= (8 + 4) ∧ (8 + 4)<= (4 + 8) ⇒ 6+0 <= 4+8

S(8) {(x/6), (y/6+0), (z/4+8)} 6 <= (6 + 0) ∧ (6 + 0)<= (4 + 8) ⇒ 6 <= 4 + 8